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All of this, on a little reflection, will probably seem obvious. But is it not strange that, in face of it, any philosopher should seriously contend that the human and personal factor in beliefs must be ignored? For it appears to be the very factor to which alone we can look to transmute the ambiguous and indeterminate pressure of the uncomprehended real into definite judgments of affirmation or denial, according to the bent of the personality engaged; thus it is the sole factor which can engender truth and render reality comprehensible. It is possible, of course, to abstract from this factor; for it appears to be irrelevant for many scientific purposes: but nevertheless depersonalization is essentially a *fiction*. It is a fiction, moreover, which conceals from our view all the subtlest and most interesting influences of vital conditions upon beliefs, and renders impossible any coherent and intelligible accounts of the relations of truth and value.

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THE LOGIC OF PROBABLE PROPOSITIONS

FORMAL logic is an analysis of principles almost instinctive to common consciousness. Quite as instinctive are the principles of probable reasoning; yet philosophers have had little success in bringing them into the high-light of criticism, and less in uniting them with formal logic. The present paper is a suggestion toward attaining this analysis and this union.

A frequent misconception of the nature of probability will serve as starting point. A judgment is probable, say Laplace and his less illustrious follower Jevons, because he who judges believes, but is ignorant of the truth or falsity of his belief. Says Jevons:[1] "Chance exists not in nature and can not coexist with knowledge; it is merely an expression, as Laplace remarked, for our ignorance. . . . Probability belongs wholly to the mind." Probability, therefore, being intensity of belief, has the same status as anger or impatience or indignation; and one may well inquire how there can be any standard of probability. The only answer is: Since one belief, *quâ* belief, is stronger than another, the probability of the proposition believed is greater on account of the feeling of conviction which attaches to the belief, not at all on account of any character attaching to the proposition believed. But there must be some standard degree of intensity of belief as point of departure. The essential (and wholly conventional) canon of this school is: Where we are ignorant a belief is as likely to be true as to be false; the probabilities of propositions of whose truth or falsity we know nothing are equal. However, no self-evidence goes with this canon. Seeing its lack of *a priori* validity and not wishing boldly to assume it, some theorists have called it a generalization from experience. Edgworth says in this connection:[2] "I submit, the assumption that any probability constant about which we know nothing in particular is as likely to have one value as another, is grounded on the rough but solid experience that such constants do as a matter of fact as often have one value as another." It is extremely doubtful that, being ignorant of the relative frequencies of events, we have learned from experience, *en bloc,* that our expectations are fulfilled about as frequently as not. Any investigation of statistics will yield thousands of cases in which the relative frequencies of events are not one to two. Had we begun in any of these cases with the equal distribution of ignorance, experience would have disappointed us by refuting our assumption. We might, quite as well, have distributed our ignorance in the ratio one to three, or one to six; and experience would, in a great many cases, have corroborated these distributions.

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[1] Jevons: *Principles of Science,* Ch. X., pp. 197–98. Also Laplace: *Théorie Analytique des Probabilities,* Introduction, p. ii.

To place probability beyond the merely conventional, we must disentangle it from belief and attach it, not to beliefs, but to propositions believed.

Any belief is true, not by virtue of the fact that it is a belief, but by virtue of some sort of relation between the real world and that which is believed. When we believe or judge, there is always something which we believe or judge. Following current usage, we may call it a proposition. In addition there is always an object of which the proposition is judged true. This object lies somewhere in the realm of fact as a "locus of verification." To believe that something is probable is not to alter this situation. There are still belief, proposition, and fact. It is as absurd to hold that comparative intensities of belief determine the comparative probabilities of propositions as that irresistible conviction determines truth. Probability, like truth, gets determined by a relation between the proposition believed probable and the state of the facts. What is this relation?

Clearly, it is not the same relation to fact as that which a true or a false proposition has. The assertion, "He will probably die of influenza," is on different level of predication from the assertion, "He died of influenza." Fact (the time element being inessential) will ultimately reduce the probable assertion to truth or falsity, with no middle ground. As a probable proposition, however, it is neither true nor false. Its probability comes from its reference to something other than the fact which will ultimately validate or invalidate it. Were fact its only referent, it must be either true or false. This other referent is *a class of propositions of which it is a member:* a class of propositions all of which make the same assertion, but which make this assertion for different instances or different situations. In the convenient phraseology of Mr. Bertrand Russell's logic, such a class of propositions may be defined as all those, and only those, propositions which result when a propositional function is given specific significant determinations of its variable.

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[2] Edgworth: "The Philosophy of Chance," *Mind,* Vol. 9, 1884, p. 223.

It is evident that the members of a class of propositions of this nature, whether the class be finite or infinite, may be all true, all false, or a part may be true and a part false, this truth or falsity being fixed by a factual "locus of verification" and not by belief. Therefore, probability is frequency of truth within such a class of propositions. Fact determines the frequency for the class. And to qualify a proposition as probable is to ascribe to it a truth frequency which holds for its class. A probable proposition has as its first referent its class; as its second and indirect referent, through its class, fact.[3]

It will be well at once to make the distinction between finite and infinite classes. Obviously, a function could be true for all, for none, or for part of an infinity of values. Therefore, an infinite class of propositions would have a truth frequency, and any single proposition of the class would be probable; with this reservation, however: the probability would not be numerical since ratio has no meaning in infinite collections. On the other hand, in a finite class of propositions the truth frequency which we call probability may or may not, as we choose, be represented numerically. It will be the ratio of all the cases in which the assertion is true to all the cases in which it can be significantly made. The obvious cases of universal truth, or "certainty," and universal falsity, or "impossibility" are to be noted in passing.

The kind of bare, abstract probability attaching to every predication which can be made separately of a number of instances or situations is the simple element from which the richer form of probable inference can be constructed. The writer uses because of its clarity Mr. Russell's notion of a propositional function. Bare numerical probability is the proportional frequency with which any propositional function will be determined as true by all values of its variable (the values being finite in number).

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[3] C. D. Broad: "On the Relation between Induction and Probability," *Mind,* N. S., Vol. 27, 1918, p. 389; views probability as an irreducible reference, a view to which the present writer can not agree. See also for a similar view, R. Demos: "A Discussion of Modal Propositions," *Mind,* same vol., p. 77.

The measurement of any specific probability is difficult. Not all facts are given or are accessible to observation. So, it is impossible to know, when a class of propositions has been defined, how many of them will be true and how many false. Some simple cases are analogous to perfect induction and present no difficulty. Consider the well-worn example of the urn containing seven white and three black balls. If the balls be drawn and not replaced, the truth frequency within the class of propositions "x is the drawing of a white ball" must be seven to ten. Suppose, however, that the balls be replaced in the urn after each drawing, so that we have an indefinitely large number of possible drawings. Now the class of propositions can never be completely compassed, for it continues to grow larger. How can the probability of drawing a white ball be measured without some assumption? Laplace assumes that the drawings are made at random, random distribution assuring the appearance of one instance as often as any other. This amounts again to the equal distribution of ignorance.

We must escape the assumption of this doubtful canon by taking the position that probability can properly be measured, not before the fact, but only after the fact. The measure of the probability of a propositional function is a limit which the truth frequency of that propositional function as actually determined by trials tends to approach. The calculation of a probability can not be *a priori;* it must always refer back to the fact in experience; hence it must always be approximate, tentative, subject to revision. The calculations of actuaries give the best examples of the proper measurement of probabilities. They are in no sense *a priori.* Probability being thus measured after the fact and being defined as a truth frequency within a class of propositions, it is not necessary that every proposition of the class be equally possible, or—what is the same thing—equally probable. It is necessary only that the propositions of the class be distinct, numerable, and finite. It remains for experience to discover the limit which its truth frequency approaches. However, discovered or undiscovered, there must be a numerical truth frequency within a finite class of propositions. The subsequent derivation of the numerical laws of probabilities rests on this statement.

Classes of propositions, obviously, may stand in logical relations to one another. They may, for example, be related by implication, disjunction, conjunction, or opposition. Further, the assertion of such a relation between classes of propositions itself defines a new class of propositions. This third class of propositions may have a truth frequency. Logical relations between propositional functions (we have called every propositional function a bare probability) give rise to the laws of probable reasoning just as they do to the laws of formal or necessary reasoning. Formal truth or necessity in such relations differs from probability in that the one holds for all values, the other only for a part of the values for which the relation can be asserted. Formal truth or necessity may in this way be regarded as a special case of probability.

The case of disjunction is fundamental in the logic of probabilities. The law for the addition of probabilities comes from it. Now, to assert that two classes of propositions stand formally in the disjunctive relation means: for any value of the variable, at least one of the assertions (or propositional functions) which define the two classes is true. This does not exclude their both being true. Such a disjunction would be: "Either a young man was willing to serve his country in the war or he was a traitor to it." This disjunction will, however, lose its formal character to become probable if there are values of the variable for which it is false as well as values for which it is true. It is false when both extremes of it are false, *i. e.*, when "a young man unwilling to serve his country is not a traitor to it." There being such young men, the assertion must be altered to a probable disjunction.

But how do the probabilities within each of the disjoined classes affect the probability of the disjunction itself? Since the disjunction is not exclusive, there may be cases in which both of its extremes are true, *i. e.*, in which "a young man is willing to serve his country and at the same time is a traitor to it." Only fact can determine how many such cases there are. However, each class of propositions, if it is finite, must contain a certain number of true propositions and a certain number of false. Each must have within it a numerical probability. If the disjunction be exclusive, so that it is true only when one or the other of its extremes is true exclusively, then the total number of cases of its truth must be the sum of the number of cases for which each of its extremes is true. The probability of the exclusive disjunction must, therefore, be expressible in terms of the probabilities within the two classes of propositions which it relates.

A consideration to be mentioned here and borne in mind throughout the discussion of the relations between classes of propositions is the identity of the variable. If the class of propositions defined by "x is A" be related to the class defined by "x is B" through the disjunction "x is A or x is B," it must be remembered that though x is a variable, it is the same variable throughout. The total number of values of the variable for which an assertion can be made forms one term (the denominator) in the probability ratio for this class of propositions; the number for which the assertion is true forms the other (the numerator). Because x is the same variable throughout, for all three classes of propositions—"x is A," "x is B," and "x is A or x is B"—the denominator of the probability ratio must be the same.

From this, the identity of the variable, and from the previous definition of an exclusive disjunction, it follows that the probability of one proposition being r/x and of another s/x, the probability of their disjunction, where they are exclusive will be $(r + s)/x$. This is the usual rule for adding probabilities.

Similarly, implication between two classes of propositions may be formal or probable. The probability of an implication, however, will bear no determinate numerical relation to the separate probabilities of its two terms. But just as strict inference depends upon the implicative relation, so does probable inference. Inference proceeds from a true proposition (as hypothesis) and a true implication. Granted the truth of the implication, "If man is humble he shall inherit the kingdom of Heaven," and granted that a certain man is humble, the inference that "he shall inherit the kingdom of Heaven" is valid. All cases of an inference are numerically represented by all cases in which the antecedent of the implication upon which it proceeds is true. (Not so with all cases of the implication; for the implication may be true when the antecedent is false. A false proposition may imply any proposition.) If the implication be formally true, the inference will also be formally true; otherwise, unless formally false, the inference will be probable. Since an inference can be made only when the antecedent of the implication has a value making it a true proposition, a probable inference is, therefore, one of a class of inferences defined as all cases in which the antecedent of a probable implication is true. Thus, a class of probable inferences is defined by all cases of humility where "humility probably implies the inheritance of the kingdom of Heaven." The total number of cases of the inference (or the denominator of the ratio specifying the truth frequently in the class of inferences) will be all the cases of the truth of the antecedent; the numerator will be all cases of joint truth of antecedent and consequent, that is, all cases in which the inference is true. This fraction, again, can not be determined by any combination of the bare probabilities of the terms of the implication with one another.

The common logical consciousness makes continual and successful use of probable inference. The notion that probability has meaning only in so far as one proposition is conditioned by some other proposition comes, it appears, from conceiving the probable only as that which is probably inferred, without analyzing this complex notion into the simpler one of bare probability in reference to a class of propositions. The foregoing analysis of probable inference is, moreover, essential to the definition of independence which conditions the rule for multiplying probabilities.

The multiplication of probabilities has its basis in the relation of conjunction. Two classes of propositions being formally conjoined, their defining assertions will be jointly true for all values of the variable. Being probably conjoined, they will be jointly true only for some values of the variable. As in the case of probable inference, their separate bare probabilities give no aid to the calculation of the probability of their conjunction. For no reason exists why the product of their separate probabilities should be the probability that they are jointly true. The number of cases of their joint truth is a matter wholly determined by fact. Suppose, however, they are independent in the sense that when one is true the probability that the other will be true remains unaffected. For example: It is probable that a man may be wounded in battle and it is also probable that he may receive the Croix de Guerre. The probability that he will receive the Croix de Guerre will remain the same whether or not he is wounded in battle. Hence, the inference "If he is wounded in battle he will receive the Croix de Guerre" is no more or less probable than the proposition "He will probably receive the Croix de Guerre." Their probability is equal. Symbolize the probability of the two original propositions by r/x and s/x. The probability that an inference can be drawn from one to the other must be, as we saw, the ratio of the number of cases of their joint truth to the total number of cases in which the antecedent from which the inference is drawn is true. If k represents the number of cases of their joint truth, then the fractions k/r and k/s will be the probabilities that an inference can be drawn from either one to the other of the propositions; for r and s are the total number of cases, respectively, in which the antecedents of the two possible inferences are true. (The identity of the variable, which makes the total number of cases of truth and falsity the same for both of the classes of propositions and also for their conjunction, must be again borne in mind. This, the denominator of the probability fraction for each class, is symbolized by x; and since x is assumed to be in its largest, or "factual," terms, r and s will be in their largest terms.) The probability of the joint truth of the two classes of propositions will necessarily be symbolized by k/x.

The independence spoken of above, which means that the probability "If a man is wounded in battle he will receive a Croix de

Guerre'' is no greater or no less than ''the probability that he will receive a Croix de Guerre,'' can be represented by the equation $r/x = k/s$. The probability of the inverse inference of independence will be $s/x = k/r$. These two equations, however, are exactly what result when we assume the ordinary rule for multiplying probabilities. If the product of the probabilities of the two propositions is equal to the probability of their joint truth, the equation $r/x \times s/x = k/x$ must be true. And this equation reduces to $r/x = k/s$ or $s/x = k/r$. Therefore, the arithmetical product of two probabilities will represent the probability that the two propositions will be jointly true on condition that the propositions are independent, in the sense that if one is true the probability that the other will be true will be unchanged.

It becomes evident from the discussion that, although specific probabilities like specific truths are to be measured by fact, the laws of combining probabilities into conjunctions, disjunctions, or inferences lie within the realm of pure logic; and that the laws of these fruitful methods of reasoning are intimately related to all other laws of thought.

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REVIEWS AND ABSTRACTS OF LITERATURE

Studies in Psychology. Contributed by Colleagues and Former Students of Edward Bradford Titchener. L. N. Wilson. 1917. Pp. 337.

This volume of studies was presented to Professor E. B. Titchener at a celebration of the completion of twenty-five years of distinguished service to Cornell University and to psychology. The book was edited by Professors W. B. Pillsbury, J. W. Baird, and M. F. Washburn, and contains contributions from twenty colleagues and former students. As there are nineteen separate reports, only the general character of each will be indicated.

W. B. Pillsbury discusses the principles of explanation in Psychology, testing them in the special case of the antecedents of action. J. M. Baird reports an experiment upon memory for absolute pitch. He finds it a capacity possessed in varying degrees by different individuals and present usually only under special conditions. Ferree and Rand present methods for measuring the ''Selectiveness of the Achromatic Response of the Eye to Wave-length and its Change with Change of Intensity of Light.'' J. N. Curtis tests the method of single stimulation, a rapid method of determining tactual dis-